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STEMSiL Handbook (Short Version)

Research in STEM Teaching and Learning in Sign Languages

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1) From Scientific Concepts to Signs: Leveraging STEM Glossaries in Deaf Education - Audrey M. Cameron

1.1 Introduction: Bridging STEM and Sign Language



This chapter examines the intersection of deaf education, sign language and STEM subjects, focusing on the SSC BSL Glossary project. It explores the development of scientific signs and their impact on conceptual understanding, demonstrating how carefully crafted signs bridge the gap between written terminology and visual-spatial cognition across various scientific disciplines. Through this exploration, the chapter highlights the transformative potential of sign language in STEM education for deaf learners.

1.2 Teaching Science and STEM Through Conceptual Understanding



Research emphasises the importance of hands-on experiments, peer dialogue, and diverse learning modalities in facilitating understanding, particularly for deaf students needing more examples to grasp concepts. Teachers play a crucial role in guiding students' construction of meaning through collaborative interpretation of experiments and activities, using probing questions to stimulate critical thinking. Deaf learners often have fewer opportunities for these experiences, highlighting the need for environments where all parties can communicate fluently in sign language. A comprehensive approach, including exploring the world outside the classroom and accessing various learning modalities, is essential for deaf children to develop a complete understanding of scientific concepts.

1.3 History and Evolution of Sign Language Glossaries



Sign language glossaries have evolved significantly over time, from static drawings and photographs in books to dynamic video content on the internet. Technological advancements, from film to digital media, have enabled more accurate representation of sign movements and easier updating of content. The establishment of STEM-specific sign language websites, pioneered by Lang at NTID/RIT, has led to significant growth in glossaries worldwide. Various approaches to creating these glossaries exist, from self-loading methods to collaborative discussions, all aimed at improving STEM accessibility for deaf people globally.

1.4 Purpose of SSC BSL Glossary



The Scottish Sensory Centre's BSL Glossary originated from Dr Mary Brennan's research in 2000, which identified a shortage of STEM signs as a significant barrier for deaf students in national examinations. Despite advancements in university support, such as the Disabled Students' Allowance (DSA) and access to interpreters, the percentage of deaf students in UK higher education STEM programmes remained stagnant at 0.3% over a decade. This lack of progress and similar findings in the US underscores the critical need for comprehensive STEM sign glossaries to support deaf students and their educators, highlighting the glossaries' importance in breaking down communication barriers in STEM education and careers.

1.5 Structure and Content of the SSC BSL Glossary



The SSC Glossary website offers a comprehensive, user-friendly interface for accessing STEM signs, organised by both topic and alphabetical order. The glossary provides video demonstrations of signs, accompanied by signed explanations and written English translations, making it a bilingual resource. In response to user feedback, the glossary also includes example videos showing real-life applications

of the signs, enhancing understanding of scientific concepts for students, educators, and interpreters alike.

1.6 Visualising STEM Concepts: The Sign Development Process



The SSC's sign development process involves a diverse team of deaf scientists, educators, and sign linguists who collaborate to create signs for STEM concepts. The team focuses on visually representing the underlying ideas of scientific terms, rather than simply translating words, through group discussions and careful consideration of each concept's visual aspects and functions. The process includes multiple stages of review, including feedback from young deaf children, before signs are finalised, filmed, and published on the SSC BSL glossary website as a bilingual resource with accompanying definitions and example videos.

1.7 Subject-Specific Sign Development: Examples from Various STEM Fields



This section explores specific examples of sign development across various STEM disciplines, including Geography, Biology, Astronomy, and Chemistry. By examining how signs are developed to represent diverse scientific concepts, from physical landscapes to abstract chemical processes, the versatility and power of sign language in conveying complex scientific ideas are demonstrated.

1.7.1 Geography: Representing Landscapes and Topography



Geography uses visual signs to represent topographical features like valleys and rivers. Contours on maps indicate slope steepness and sign shapes change to depict various landscapes.

1.7.2 Biology: Visual Representation of Location and Function



In developing signs for biology, careful consideration is given to accurately represent the location, shape, and function of biological structures, such as organs and plant parts. The process involves extensive research, consultation with experts, and sometimes revision of signs to ensure precision and accuracy, as demonstrated by the development of the sign for 'stem cell'.

1.7.3 Astronomy: Designing Planetary Signs



In developing signs for planets in the solar system, the team focused on both visual aspects and unique properties of each celestial body. The signs incorporate distinctive characteristics such as Mercury's temperature extremes, Venus' dense atmosphere, Mars' two moons, and Neptune's uniquely orbiting moon, creating a cohesive set of signs that maintain connections between planets while highlighting their individual features.

1.7.4 Chemistry – Representing Abstract Concepts



Chemistry, being abstract and theoretical, presents unique challenges in teaching, often involving invisible processes at the molecular level. The development of signs for chemistry concepts, such as atoms, electrons, and chemical reactions, provides visual representations that help bridge the gap between macroscopic observations and microscopic processes, aiding in the understanding of complex chemical concepts across Johnstone's Triangle framework of macroscopic, microscopic, and symbolic levels.

1.7.5 Family of Signs: Aiding Conceptual Understanding



The sign development team created 'families' of signs to build a comprehensive understanding of broader scientific concepts. For example, the signs for 'mass', 'gravity', and 'weight' are interconnected to demonstrate their relationship in physics. Similarly, in chemistry, a family of signs related to chemical reactions maintains consistent elements while introducing variations to represent different types of reactions, enhancing conceptual understanding through visual connections.

1.8 Impact of Sign Glossaries on Learning

1.8.1 Conceptual Understanding: Electricity - AC vs DC concept



A research project observing classroom discussions revealed how access to the STEM Sign Glossary can significantly aid conceptual understanding. In a presentation on AC and DC electricity, a student's initial confusion about the term 'current' was resolved when introduced to the correct scientific sign, demonstrating the glossary's importance in clarifying scientific concepts and preventing misunderstandings due to everyday language use in scientific contexts.

1.8.2 Vocabulary Access: Teaching Density



In a lesson on floating and sinking for young deaf pupils, the use of the sign glossary, particularly the sign for 'density', enhanced conceptual understanding. After a hands-on activity and explanation using the signs, the children were able to correctly attribute floating and sinking to the concept of density, demonstrating how sign language vocabulary can effectively convey complex scientific concepts even to young learners.

1.8.3 Facilitating Understanding Through Sign and Dialogue



Lindahl's research (2021) highlights that while sign language, text, and pictures are crucial for conceptual understanding in deaf education, effective learning requires more than just vocabulary access; it necessitates facilitated discussions and educators who can recognise and respond to students' use of signs that demonstrate comprehension.

9.1 Conclusion



The development of sign language glossaries for STEM subjects represents a significant advancement in deaf education by creating visual representations that capture the essence of scientific concepts. These STEM signs, developed through a process involving deep consideration of scientific principles and sign linguistic structures, make abstract concepts more tangible and accessible across various disciplines. As we move forward, continued research and collaboration between deaf scientists, educators, and linguists will be crucial in integrating these resources into STEM curricula, ensuring equal access to scientific knowledge for deaf students and enriching the field of science communication as a whole.

2) The Development of Mathematical Skills of Deaf Learners: Insights from Research and Examples from Practice.

- Olga Pollex, Swetlana Nordheimer, Viktor Werner

2.1 Introduction

The authors of the paper come from different theoretical traditions and use different research methods in our scientific work. This article should therefore be understood as a multi-perspective dialogue. Research on the topic of mathematical development and teaching of deaf children has diverse theoretical, empirical and practical approaches to the education of deaf learners. In the last 20 years in particular, more research has been carried out into mathematical development of deaf children. This is reflected in the increasing number of publications on the subject. To demonstrate this variety, this article refers to the theoretical approaches and empirical studies to discuss arguments for teaching mathematics in sign languages.

In line with the diversity of positions, theoretical backgrounds, and practical intentions in mathematical education of deaf schoolchildren, the terms “deaf” or “hard of hearing” are not used consistently in the scientific literature (see Szücs, 2019). To avoid misunderstandings, we will use the term “deaf” as in Scott, Henner and Skyer (2023) “to refer to a range of hearing levels, from what might typically be referred to as hard-of-hearing, to profoundly deaf; we also include anyone who would benefit from being identified as deaf such as those with central auditory processing disorder, as we believe that all would benefit from the model proposed here.”

2.2 Theoretical Framework

Rosanova (1971) and Yashkova (1971) empirically showed that deaf children are multilingual and that different language systems are complexly linked in their thinking. Studies conducted by Villwock et. al. (2021) give differentiated, deep and empirically grounded insights into the complexity of activation of different languages by hearing and deaf ASL-English bilinguals when they process written words. However, Rosanova (1991) assumed that language competences alone do not determine successful development of mathematical abilities in deaf learners. She suggested integrating the fostering of ‘**visual-imaginative**’ thinking and ‘**logical-verbal**’ thinking in order to teach mathematics successfully.

Visual-imaginative thinking is the ability to think in images and representations that replace real objects in order to carry out mental operations. Both the external appearance, and the properties of objects and the relationships between them should be taken into account in visual-imaginative thinking. To this end, Rosanova (1978) recommends strengthening the relationships between objects

and words that denote the objects, their properties and relationships. We extend this recommendation and further suggest that the development of visual-imaginative thinking can be mediated, guided, supported, and strengthened by using **productive** and **conventionalized signs** and **gestures** as designations mathematical objects themselves, their properties, and the relationships between them.

Logical-verbal thinking involves formal mental operations mediated through language that may be completely detached from real objects. Here too, we go further than Rosanova (1978) and suggest that this form of thinking should also be consciously embedded in sign languages as early as possible in order to provide optimal teaching and support.

2.3 Empirical Findings

Recent empirical findings support the positive effects of sign languages on the mathematical development of deaf children. We will now summarize some of the arguments derived from empirical studies.

- Leybaert and Van Cutsem (2002) investigated to what extent the visual-manual modality and the structure of the sign number sequence has an influence on the development of counting and its use by deaf children.

- Di Luka and Presenti (2011) argue that finger-numbers help to acquire, build, and then access number semantics, and that they provide additional value compared to other number representations by anchoring the meaning of numbers in a self-experienced sensory-motor representation. However, finger-numbers are not signed numbers or signed algorithms which belong to the national sign languages.

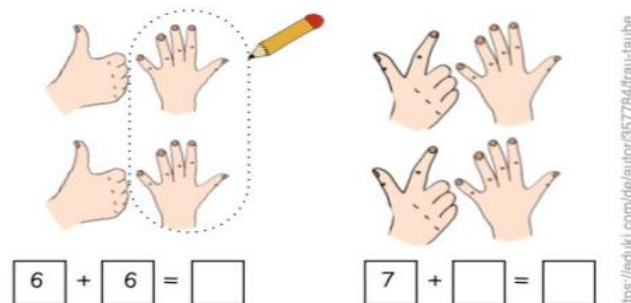
- Werner and Hänel-Faulhaber (2023) investigated deaf and hearing children's understanding of repeating patterns. In these tasks, the children had to fill in a gap in the patterns. It was found that the solution scores of deaf children who learned sign languages at an early age were comparable to those of hearing children. In contrast, deaf children who learned sign languages later were less successful. This shows that sign languages have a positive effect on pattern solving tasks.

- An evaluation of a version of the mathematical diagnostic test MBK 0 (a test of basic mathematical skills at kindergarten age; Krajewski, 2018) in German Sign Language found that the results of six-year-old deaf native signers correspond to the (hearing) age norm (Werner & Hänel-Faulhaber, 2024).

2.4 Practical Examples

Finally, we substantiate our considerations with the practical examples provided by Olga Pollex. These are intended to serve as an additional source of argumentation for sign languages and signed mathematics on the one hand, and as a stimulus for the development of didactic concepts and materials on the other. Using concrete examples from geometry, arithmetic and combinatorics, she explains how mathematical concepts, theorems and proofs can be introduced through actions such as operations with geometric figures made of paper. She then suggests using productive signs derived from actions to describe the processes.

Once students understand the concepts, conventionalized mathematical signs and their lexical variations can be introduced. The principle of this increase in sign language use is based on the EIS-principle according to Bruner: the integration of enactive, iconic and symbolic modes of representation. Mathematical teaching is often reduced to the promotion of arithmetic skills and the promotion of process-related skills is omitted. This reduction is partly related to the sign language skills of learners and teachers. However, in order to recognize mathematical relationships, to link knowledge and skills and to transfer them to unknown questions, language is needed.



[IS video](#)



[DGS video](#)

3) Deafdidaktik-critical View of mathematical Text Tasks

- Bastian Staudt, Horst Sieprath, Ege Karar, Merve Baklaci, Daniel Schmidt, Klaudia Grote

In the context of an empirical study on DeafDidaktik by Staudt (2024) in mathematics lessons with deaf students whose first language (L1) is German Sign Language (GSL), it was repeatedly observed that working on mathematical text tasks is associated with specific comprehension difficulties.

These issues were discussed and analyzed with the DeafDidaktik-Team at the SignGes Competence Centre for Sign Language and Gesture at the RWTH Aachen University under the direction of Dr. Klaudia Grote. Based on these considerations, a DeafDidaktik adaptation of a text task designed for hearing children was developed, resulting in a mathematical text task tailored to the needs of deaf children. To assess the efficacy of the adapted task, a preliminary empirical study was conducted, in which a 'kangaroo task' was presented to two ten-year-old children: one hearing and one deaf.

Note: The so-called 'kangaroo tasks' are an example of a pedagogical approach that originated in the Australian education system and has since been adopted in a European mathematics competition. The kangaroo tasks have been employed in Australian educational institutions since 1978, with their implementation in German schools following 4 years later. The objective of these tasks is to provide support and challenge for students in the third and fourth grades regarding mathematical learning (for further information, see <https://www.mathekaenguru.de/international/index.html>—09.11.2024).

The text task from the 2021 Kangaroo Competition, which Staudt introduced to the schoolchildren in a preliminary study, is as follows: **In a modest cinema, five companions occupy an entire row. Paul is not seated in the fifth position. Anabel, on the other hand, has selected the first seat. Lynn is situated between Joshua and Selin. Thus, the question arises as to the precise location of Lynn's seating.**

The hearing child with German as their first language (L1) solved the task promptly and accurately. The deaf child, whose first language is German Sign Language (DGS), acquired at a relatively late stage, and German, which may be considered a second or even third language acquisition due to the Russian migration background of the parents, experienced considerable difficulties in reading and understanding the above text task. Subsequently, the child was presented with a translation of the task in DGS. The child's feedback and reactions suggest an enhanced comprehension of the signed task. Nevertheless, despite the translation into DGS, the child could still not comprehend the mathematical methodology for completing the task.

Subsequently, the child was presented with an animation of the content, designed following the principles of DeafDidaktik and edited with the software PowerPoint. Immediately following the initial presentation, the child demonstrated an understanding of the context depicted. In a second iteration, a brief signed explanation of the task was additionally provided, facilitating comprehension of the mathematical approach and enabling the child to complete the task.

For the DeafDidaktik-version of the text task, DeafDidaktik-principles had to be applied, which required a three-phase DeafDidaktik-analysis in advance. The final presentation of the material included videos in German Sign Language (DGS) and PowerPoint slides with corresponding animations and transitions, each incorporating Principles. These included an inductive style of explanation, subject-object buoys, a signed elimination strategy, localization, and changes in perspective. This was achieved by applying sign classifiers and, in addition, constructed action (CA) or constructed dialogue (CD) (Grote, Sieprath, Staudt, Fenkart & Karar – Work in Progress 2024). Furthermore, elements of DeafScience were incorporated, including the presentation of sign language videos in circular formats with color-coded frames to differentiate between them. In this case, the color 'white' represents the introduction of the task, 'blue' represents additional explanations, 'red' represents the question, and "green" represents the answer or solution (Sieprath et al., 2024).

This preliminary study indicates that deaf students encounter various challenges when solving mathematical tasks in a written form. These text tasks require the students to employ a variety of decoding procedures or processes, including decoding the content, translating the written text into mathematical codes, and solving the mathematical problem.

Considering the findings of this preliminary study, this video presents the initial criteria for creating signed DeafDidaktik videos for mathematical tasks. However, it is essential to note that these criteria require further empirical investigation in educational contexts.



[IS video](#)



[DGS video](#)

4) Signing about Variables and Equations

- Flavio Angeloni, Christian Hausch

Bilingual practice with a sign language and a written language is fundamental in teaching sign language-oriented pupils. However, the characteristics of sign languages should also be considered in education research. Over time studies have shown that sign languages can influence the teaching and learning of mathematics in such a way that significant differences to spoken language practice can sometimes arise. It has also already been shown, for example, that “[...] the use of sign language space in the mathematics classroom can have a decisive function, e.g. [...] in the acquisition of specialist and technical sign language signs that do not (only) consist of certain signs for specialist and technical terms of [a] spoken language” (translation from Angeloni, 2023, p. 532).

In this chapter, basic notions and concepts of elementary algebra – such as “variable”, “equation”, etc. – are considered from a sign language perspective based on the results of a broader project on teaching and learning elementary algebra in a sign language. In the first section, the investigated variable aspects – object aspect, substitution aspect and the shell aspect – and a central property of sign languages – iconicity – are presented as they relate to the field of mathematics. Iconic Signs are signs that show a direct or indirect similarity to the reference. In the first case, the signs are defined as *pictorial*, in the second case as *schematic icons*. Both types of iconic signs are divided into different subtypes. Then, we address the key principles for teaching in a sign language, as well as the learning environments used throughout the duration of the project. For bilingual mathematics classrooms which employ a sign language, “various modality-related structural differences between spoken and signed languages should be taken into account” (translation from Grote et al., 2018, p. 435). This includes a higher degree of iconicity in sign languages than in spoken languages, due to which there is a stronger coherence between the signs and the properties of the reference. In mathematics classes, therefore, “the type of explanations should correspond with the iconic aspects [...]” (translation from Grote et al., 2018, p. 433).

Another characteristic is *centering*: A topic is placed at the center and a syntagmatic context is established. This means that different concepts that are used together are placed in relation to each other around a central concept. A syntagmatic context can be created by changing the perspective, i.e. switching from one specific concept to another in order to describe it in more detail. This would mean in the mathematics class, for example, that a specific central topic should be placed at the center around which further knowledge units are placed.

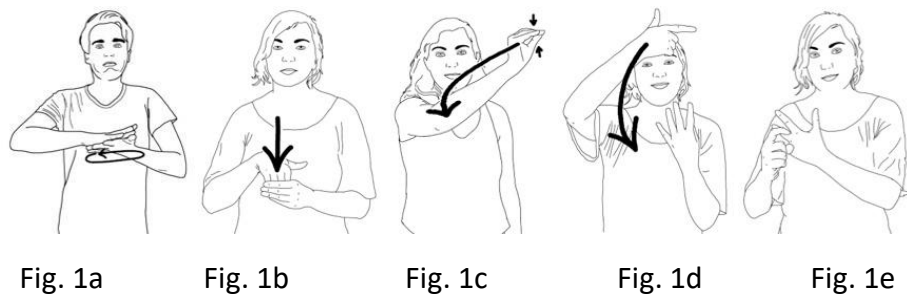


Figure 1: Signs EMPTY (left) and CONTENT (right)

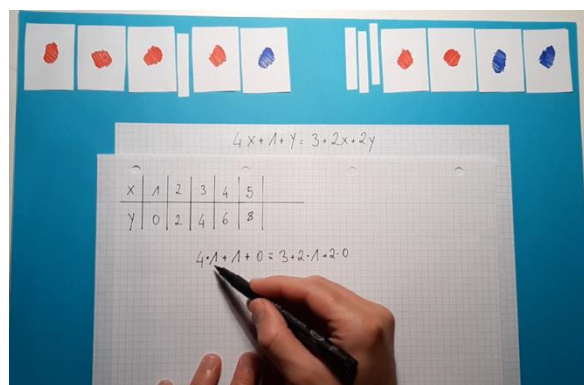


Figure 2: Excerpts from the videos that are used as tasks

Selected results on the object aspect, the substitution aspect and the shell aspect of variables are explained and possible implications for mathematics instruction are discussed. The presented signs belong to Austrian Sign Language (ÖGS). The results show that there are different signs that can be used to communicate about variables and related actions. The signs in Figures 1c and 1d are pictorially iconic because they are directly similar to the written image with the table of values above the equation (Fig. 2). In addition, the sign in Figure 1c imitates an action in which something is taken from the table and is placed where the equation is. The sign EMPTY (Fig. 1a) is specifically interesting, because it communicates that the variable is empty, it has no numbers. Therefore, if the sign CONTENT (Fig. 1b) – according to that a variable would be regarded as a container – is considered in relation to the sign EMPTY, then the unknown nature of the number would be expressed in the form of an empty container (an empty variable), which places the shell aspect in the foreground. This aspect can also be observed in the case of the substitution aspect, for example by the sign construct in Figure 1e. Assuming that the shell aspect plays a key role, this could implicate for the practice of mathematics education that the shell aspect could be seen as a “central” variable aspect around which further variable aspects can be located and thus all aspects can be placed in a syntagmatic relationship to each other.



[IS video](#)



[DGS video](#)

5) An example of a Task from Stochastics with German Sign Language (DGS) - Elke Warmuth, Swetlana Nordheimer, Tino Sell

This article presents an example from stochastics which is connected to geometry. By choosing a geometric context, we want to address the strengths of deaf learners known from the specialized literature and to facilitate their access. The problem presented in this article is a modification of the 4th task of the 5th example in the educational standards at primary level (KMK 2004, p. 20). It is mainly assigned to the key idea of *space and form* can be found in requirement area III. The first modification of the task was made by Winkenbach (2011). We have didactically and linguistically revised the task one more time to offer it bi-modally in German Sign Language (DGS) and in German. The German version of the tasks refers to the edge-model (see Fig. 1-3). It is presented below:

This is an edge model of a cube. A beetle is sitting in corner A. It wants to crawl to corner Z. The beetle's eyes are blindfolded. The beetle feels its way along the edges of a cube. The beetle stops at each corner and makes a lucky turn. But it never goes back. After three edges, the beetle is tired and stops. In the picture, the beetle first goes along the red edge. If it turns onto the light blue edge at the end of the red edge and onto the pink edge at the end of the light blue edge, it will arrive at Z. However, if it crawls in the red-black-orange order, it will not arrive at Z after three edges. How many paths are possible? What is the probability that the beetle will reach its destination?

In Figures 1 to 3, the excerpts from the sign language representations of an example of a favorable and an unfavorable path are linked to the edge model and the corresponding position of the beetle in the model space or in the corners of the cube.



Figure 1: Favorable path
"The beetle has reached its destination"



Figure 2: Unfavorable path
"Target missed"



Figure 3: The sign for
"probability" or "chance"

With deaf learners in mind, we think it makes sense to provide colored edges in the model or drawing. In variant A for children who are beginning to learn DGS, a physical edge model is shown and all the sentences are demonstrated on the edge model. In variant B for children who have a good command of DGS, the physical model is not shown but only signed. The picture may be superimposed.

To demonstrate the concept behind the problem solution, we recommend drawing a tree diagram (Fig. 4) that visualizes the beetle's decisions at each corner. At starting point A, it can choose green, blue-dashed or red. If it has chosen green and crawled along this green edge, it is faced with the choice of blue or orange, as it will not crawl back. If it has now crawled along the orange edge, the last choice is purple or black. If it chooses purple, it arrives in Z. Its path can be seen on the third cube from the top. If it chooses black, it misses its destination, as the fourth cube shows.

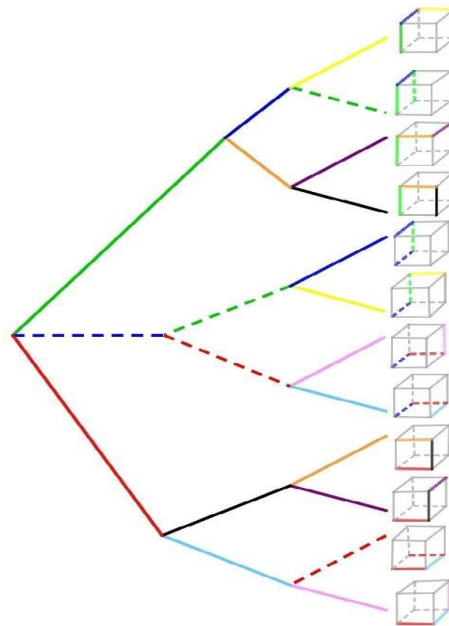


Figure 5: Tree diagram for the beetle task

The tree diagram is an important tool in combinatorics and probability theory. It structures and models the real situation and systematically records all cases. Like any visualization tool, a tree diagram must be acquired by learners. The colors in the tree diagram correspond to the colors of the edges of the cube.



[IS video](#)



[DGS video](#)

6) Sign Geometry

- Swetlana Nordheimer, Tino Sell

This paper focuses on teaching Signed Geometry to deaf learners and gives a concrete teaching example for a **volume of a cube**. Leaning on the theoretical framework of the mathematical abilities of deaf learners established by Rosanova (1991), we suggested paying more attention to the development of **verbal-logical** and **visual-imaginative** thinking of deaf children as an interplay of components. Going further and based on more recent empirical findings we suggest the conscious and targeted implementation of sign languages into geometry classes.

The empirical findings on teaching geometry to deaf learners can be summarized as follows:

- The performance on the Spatial Relations task is related to the deaf participants' language ability in their preferred modality (sign or spoken language). Using and fostering the preferred mode of communication and instruction appears to be more relevant than focusing only on the specific visualization of mathematical ideas.
- Sign languages appear to be not only the preferred mode of communication for many deaf school children, but also the tool which helps them to perceive geometrical objects, to memorize concepts and to solve problems.
- Geometry seems to be an area of strength for deaf children and can be used as a tool to teach other mathematical areas like arithmetic.
- To teach geometry, it is important to connect active operations with models and visualizations embedded in the language to help students to produce not only concrete pictorial but also schematic images of the abstract geometrical concepts.
- Spatial geometry seems to play a crucial role in the mathematical development of children.

In line with the theoretical framework and empirical findings we suggest giving deaf students not only opportunities to create visual images through signing but also to find ways for the controlled use of signed geometrical visualizations. In geometry classes, this can be achieved in three ways.

- Firstly, it may be helpful to make it explicit to the learners when the image or visualization represents one concrete example or when its aim is to visualize general propositions.
- **Conventionalized** and **productive signs** and gestures could be used as instruments for the controlled use of geometrical images to solve mathematical problems.
- Variations of geometrical visualizations and the building of **geometrical patterns** from many different cases can be helpful to teach deaf learners how to generalize geometrical visualizations and to derive general propositions by studying many cases and comparing them with each other. In this paper, an example will be given showing how this could be achieved in geometry lessons when referring to the volume of a cube.

To connect theoretical ideas and practical recommendations for teaching geometry, we illustrate them with a teaching example. It is of great significance that sign language connects the models and visualizations from the very beginning and contributes to the development of concrete, but also schematic images.

Operating with schematic images can contribute to the development of abstract ideas. There are various approaches that may be applied to the subject. For example, teachers could cut or saw cubes of plasticine, cheese, soap or even wood into smaller cubes together with the learners in craft lessons. It is crucial that the actions are introduced by signs and gestures derived from the actions on the one hand and documented by videos, models and pictures on the other.



Figure 1: $16+16+16+16$



Figure 2: $4 \times 4 \times 4 \times 4 = 64$

We suggest starting with a wooden cube with a volume of 64 cm^3 to introduce the geometric idea of the volume on the one hand and to visualize a concrete cubic number 64 on the other. The number or the value can be interpreted as a result of three times multiplication of the same number 4. The concrete value for the volume can be interpreted as the amount of little cubes in the height, length and width of the cube. It can be also divided into four portions of 16 little cubes.

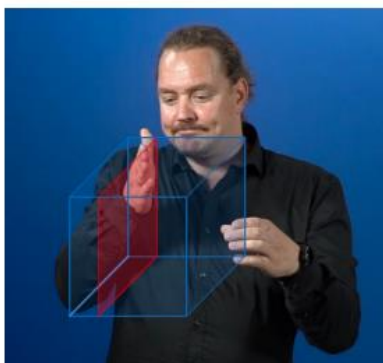


Figure 2: Cutting of a cube by parallel planes in German Sign Language (DGS)

After that, the problem can be represented in sign language by the teacher who can also ask the students to sign and to give them the possibility not only to see the cube in the wooden model or in the visually perceived signs, but also to “feel” and form it in their hands by repeating the signs

themselves. They can operate with an invisible cube by cutting it into smaller cubes and re-assemble their signs with a wooden model again. To discover the pattern and to generalize the concept the signed description of the cube can be repeated and varied with other cubic numbers like 8, 27 or even 1000.

To analyze the geometrical structure of the problem in depth, it could be helpful to integrate drawing or GeoGebra sketches into signed versions of the problem or work with several screenshots presented as picture-story of the problem (see Figure 2). Closing our considerations, we invite teachers as experts in the field to provide us with critical comments and create more and new examples which consider specific talents and needs of the concrete learners.



[JS video](#)



[DGS video](#)

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